Int. J. Heat Mass Transfer. Vol. 20, pp. 807–814. Pergamon Press 1977. Printed in Great Britain

COMBINED FREE AND FORCED CONVECTION FLOW ABOUT INCLINED SURFACES IN POROUS MEDIA

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(*Received 5 July 1976 and in revised form 24 September 1976*)

Abstract—The problem of combined free and forced (mixed) convection about inclined surfaces (or wedges) in a saturated porous medium is analyzed on the basis of boundary-layer approximations. Similarity solutions are obtained for the special case where the free stream velocity and wall temperature distribution of the inclined surface vary according to the same power function of distance. Both aiding and opposing flows are considered. It is found that the parameter governing mixed convection from inclined surfaces in porous media is Gr/Re. Numerical solutions are obtained for mixed convection from an isothermal vertical flat plate as well as an inclined plate with constant heat flux, having an inclined angle of 45°. Temperature and velocity profiles for these two cases at different values of Gr/Re are presented. For aiding flows the heat-transfer rate is shown to be asymptotically approaching the forced or free convection values as the value of Gr/Re approaches the limits of zero and infinity. The criteria for pure and mixed convection from inclined surfaces in porous media are established.

NOMENCLATURE

A, constant defined in equation (6b);
B, constant defined in equation (9);
C, specific heat of the convective fluid;
f, dimensionless stream function defined by equation (15);
Gr, local Grashof number, $Gr \equiv |g_x||T_w - T_\infty|\beta K x/\nu^2$;
g, acceleration due to gravity;
g_x, g_y, gravitational acceleration in x and y directions;
h, local heat-transfer coefficient;
K, permeability of the porous medium;
k, thermal conductivity of the saturated porous medium;
m, angle parameter, $m = 2n/(n+1)$;
n, constant defined in equation (9);
Nu, local Nusselt number, $Nu = hx/k$;
p, pressure;
Pr, Prandtl number, $Pr \equiv \nu/\alpha$;
q, local heat-transfer rate;
Ra, modified local Rayleigh number, $Ra \equiv \rho_\infty |g_x|\beta K|T_w - T_\infty|x/\mu\alpha$;
Re, local Reynolds number, $Re \equiv U_\infty x/\nu$;
T, temperature;
U_∞, free stream velocity in x-direction;
u, Darcy's velocity in x-direction;
v, Darcy's velocity in y-direction;
x, coordinate along the inclined impermeable surface;
y, coordinate perpendicular to the inclined impermeable surface;
z, coordinate parallel to the gravitational acceleration.

Greek symbols

α, equivalent thermal diffusivity;
β, coefficient of thermal expansion;
δ_T, thermal boundary-layer thickness;
η, dimensionless similarity variable defined in equation (14);
η_T, value of η at the edge of the thermal boundary layer;
θ, dimensionless temperature defined by equation (16);
λ, constant defined in equation (6b);
μ, viscosity of convective fluid;
ν, kinematic viscosity of the convective fluid;
ρ, density of convective fluid;
ϕ, velocity potential;
ψ, stream function.

Subscript

∞, condition at infinity;
w, condition at the wall.

INTRODUCTION

DURING the past decade much work has been done on the study of combined free and forced (mixed) convection boundary-layer flow about inclined surfaces immersed in a viscous fluid. For the wedge configuration, similarity solutions are obtained by Sparrow *et al.* [1] for the special case where the wall temperature and the wedge angle are varying according to a particular manner. A series solution, valid for arbitrary values of wedge angle and wall temperature distribution, is later obtained by Gunness and Gebhart [2]. For the problem of mixed convection from a vertical flat plate where similarity solutions are not possible, solutions have been obtained based on the integral method [3], per-

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turbation method [4–6], local similarity method [7], and numerical method [8].

The corresponding problem of mixed convection in a porous medium has important applications in geothermal reservoirs where pressure gradients are generated as a result of withdrawal or reinjection of geothermal fluids. It appears that the first paper on the study of combined free and forced convection in a porous medium is due to Combarnous and Bia [9] who had studied the effect of mean flow on the onset of stability in a porous medium bounded by two isothermal parallel plates. Numerical solutions are later obtained by Horne and O'Sullivan [10], Cheng and Lau [11], and Cheng and Teckchandani [12] to study the effects of withdrawal of fluids in a hot-water geothermal reservoir. Most recently, Schrock and Laird [13] have performed an experimental study on the simultaneous withdrawal and injection of fluids in a porous medium.

In this paper we shall study the combined free and forced convection boundary-layer flow along inclined surfaces embedded in porous media. It is found that similarity solutions exist when both the wall temperature distribution of the plate and the velocity parallel to the plate outside the boundary layer vary according to the same power function of distance, i.e. x^{λ}. The value of Gr/Re is found to be the controlling parameter for the mixed convection from inclined plates in a porous medium. Numerical solutions are obtained for mixed convection from an isothermal vertical flat plate (i.e. $\lambda = 0$) as well as an inclined plate with constant heat flux, having an inclined angle of 45° (i.e. $\lambda = 1/3$). The criteria for pure and mixed convection from inclined surfaces in a porous medium are established.

ANALYSIS

Consider the problem of combined free and forced convection about a wedge with an included angle $m\pi$ (or a plate inclined at an angle $m\pi/2$ with respect to

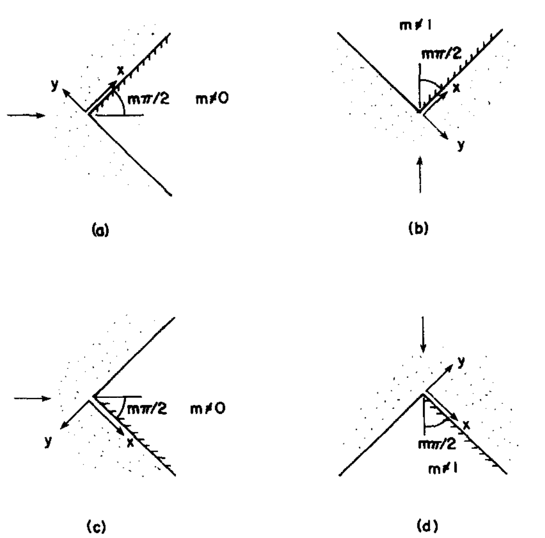

FIG. 1. Coordinate systems.

the horizontal direction) in a porous medium as shown in Fig. 1, where x and y are the Cartesian coordinates in the direction along and perpendicular to the inclined surface under consideration. In the mathematical formulation of the problem, we shall assume that (i) the convective fluid and the porous medium are everywhere in local thermodynamic equilibrium, (ii) the temperature of the fluid is everywhere below boiling point, (iii) properties of the fluid and the porous medium are homogeneous and isotropic, and (iv) the Bousinesq approximation is invoked. Under these assumptions, the governing equations for the problem are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u = -\frac{K}{\mu}\left(\frac{\partial p}{\partial x} \pm \rho g_x\right), \tag{2}$$

$$v = -\frac{K}{\mu}\left(\frac{\partial p}{\partial y} \pm \rho g_y\right), \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right), \tag{4}$$

and

$$\rho = \rho_\infty[1-\beta(T-T_\infty)], \tag{5}$$

where the "+" signs in equations (2) and (3) are for the coordinate systems shown in Figs. 1(a) and (b) while the "−" signs are for those shown in Figs. 1(c) and (d); u, v, $g_x \equiv g\cos m\pi/2$ and $g_y \equiv g\sin m\pi/2$ are the components of velocity and gravitational acceleration vectors along the x and y directions; ρ, T, p, μ, and β are the density, temperature, pressure, viscosity, and the thermal expansion coefficient of the fluid; K is the permeability of the saturated porous medium; $\alpha \equiv k/(\rho_\infty C)_f$ is the equivalent thermal diffusivity with k denoting the thermal conductivity of the saturated porous medium and $(\rho_\infty C)_f$ the product of the density and specific heat of the fluid. The subscript "∞" in equation (5) denotes the condition at infinity.

The boundary conditions for the problem are

$$y = 0, \quad v = 0, \quad T_w = T_\infty \pm Ax^{\lambda}, \tag{6a, b}$$

$$y \to \infty, \quad u = U_\infty, \quad T = T_\infty, \tag{7a, b}$$

where $A > 0$. We will designate as aiding flows when the buoyancy force has a component in the direction of free stream velocity, i.e. $T_w = T_\infty + Ax^{\lambda}$ in Figs. 1(a) and (b), or $T_w = T_\infty - Ax^{\lambda}$ in Figs. 1(c) and (d). On the other hand, we will designate as opposing flows when the buoyancy force has a component opposite to the free stream velocity such as the case with $T_w = T_\infty - Ax^{\lambda}$ in Figs. 1(a) and (b) or $T_w = T_\infty + Ax^{\lambda}$ in Figs. 1(c) and (d).

Analogous to the classical boundary-layer theory, we shall separate the saturated porous medium into two regions: (i) the boundary-layer region (or inner region) adjacent to the inclined surface where density gradient of the fluid exists and convection takes place, and (ii) the region away from the inclined surface (or the outer region) where density of the fluid can be considered to be constant. Thus, for the outer region, we can rewrite equations (2) and (3) as

$$u = -\frac{\partial \phi}{\partial x} \quad \text{and} \quad v = -\frac{\partial \phi}{\partial y}, \tag{8}$$

with $\phi \equiv (K/\mu)(p \pm \rho g z)$ denoting the velocity potential where z is the coordinate in parallel with the gravitational acceleration vector and therefore $\partial z/\partial x = \cos m\pi/2$ and $\partial z/\partial y = \sin m\pi/2$. Substituting equation (8) into equation (1), we have

$$\nabla^2 \phi = 0,$$

which is the Laplace equation. From potential theory, we known that the velocity in the x-direction along the inclined surface for the coordinate shown in Fig. 1 is given by

$$U_\infty = Bx^n, \tag{9}$$

where $B > 0$ and n and m are related by $m = 2n/(n+1)$ or $n = m/(2-m)$.

We now turn our attention to the region adjacent to the inclined surface where density gradient of the fluid exists (i.e. the inner region). If we introduce the stream function such that $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, the governing equations (1)–(5) in terms of ψ and T are

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \pm \frac{\rho_\infty \beta K}{\mu}\left(g_x \frac{\partial T}{\partial y} - g_y \frac{\partial T}{\partial x}\right), \tag{10}$$

and

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha}\left(\frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y}\right), \tag{11}$$

where the "+" sign is for Figs. 1(a) and (b) while the "−" sign is for Figs. 1(c) and (d). If convection takes place in a thin layer such that $\partial/\partial x \ll \partial/\partial y$, it follows that (i) the first terms on the LHS of equations (10) and (11) are small in comparison to their second terms, and (ii) the second term on the RHS of equation (10) is small in comparison with the first term provided that g_x and g_y are of the same order of magnitude. The latter approximation is valid for a wide range of inclined angles except for $m = 0$ in Figs. 1(a) and (c) or $m = 1$ in Figs. 1(b) and (d), i.e. for horizontal boundary layers where $g_x = 0$. With boundary-layer approximations, equations (10) and (11) are given by

$$\frac{\partial^2 \psi}{\partial y^2} = \pm \frac{\rho_\infty \beta K g_x}{\mu}\frac{\partial T}{\partial y}, \tag{12}$$

and

$$\frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha}\left(\frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y}\right), \tag{13}$$

where the "+" and "−" signs correspond to the upper and lower figures in Fig. 1. It should be noted that the boundary condition for the velocity in the x-direction at the edge of the boundary layer must be matched with the velocity given by equation (9).

To seek similarity solutions for equations (12) and (13) with boundary conditions (6), (7), and (9), we introduce the following dimensionless similarity variables

$$\eta = \left(\frac{U_\infty x}{\alpha}\right)^{1/2}\frac{y}{x}, \tag{14}$$

$$\psi = (\alpha U_\infty x)^{1/2} f(\eta), \tag{15}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}. \tag{16}$$

In terms of the new variables, it is easy to show that the velocity components are given by

$$u = U_\infty f'(\eta), \tag{17}$$

$$v = \frac{1}{2}\left(\frac{\alpha U_\infty}{x}\right)^{1/2}[(1-n)\eta f' - (1+n)f], \tag{18}$$

and the governing equations (12) and (13) become

$$f'' = \pm \frac{|g_x|\beta A K \nu}{B} x^{2(\lambda - n)}\theta', \tag{19}$$

$$\theta'' = \lambda\theta f' - \frac{1+n}{2} f\theta', \tag{20}$$

where the primes in the equations are the differentiation with respect to η and the positive and negative signs in equation (19) denotes aiding and opposing flows respectively.

In terms of new variables, boundary conditions for equations (19) and (20) are

$$\eta = 0, \quad f = 0, \quad \theta = 1, \tag{21a, b}$$

$$\eta \to \infty, \quad f' = 1, \quad \theta = 0. \tag{22a, b}$$

It is apparent that equations (19)–(22) will be independent of x if $n = \lambda$ in equation (19). Under this restricted condition, equations (19) and (20) become

$$f'' = \pm \frac{Gr}{Re}\theta', \tag{23}$$

$$\theta'' = \lambda\theta f' - \frac{1+\lambda}{2} f\theta', \tag{24}$$

where

$$\frac{Gr}{Re} = \frac{|g_x||T_w - T_\infty|\beta K x/\nu^2}{U_\infty x/\nu} = \frac{|g_x|A\beta K}{B\nu},$$

which is the ratio of the modified Grashof number and the Reynolds number.

With the aid of equation (22), equation (23) can be integrated once to give

$$f' = \pm \frac{Gr}{Re}\theta + 1. \tag{25}$$

Equations (24) and (25) with equations (21) and (22) are the governing equations and boundary conditions for the problem of combined free and forced convection about a plate inclined with the horizontal direction at an angle $m\pi/2$ [where $m \neq 0$ in Figs. 1(a) and (c), and $m \neq 1$ in Figs. 1(b) and (d)] with a wall temperature distribution given by $T_w = T_\infty \pm Ax^\lambda$, embedded in a porous medium with free stream velocity given by $U_\infty = Bx^\lambda$ where $m = 2\lambda/(1+\lambda)$.

The quantity Gr/Re in equation (25) is a measure of relative importance of free to forced convection, and is the controlling parameter for the present problem. Let us now examine the limiting case of $Gr/Re \to 0$, i.e. forced convection about an inclined plate with $T_w = T_\infty \pm Ax^\lambda$ and $U_\infty = Bx^n$. For the special case of $Gr/Re = 0$, equations (19)–(20) show that similarity solutions are possible for arbitrary values of λ and n. For this limiting case, equation (19) can be integrated to give

$$f' = 1 \quad \text{and} \quad f = \eta, \tag{26a, b}$$

where we have made use of boundary conditions (22a) and (21a). Substituting equations (26) into equations (15), (17), and (18), we have

$$\psi = U_\infty y = Bx^n y, \tag{27}$$

$$u = U_\infty = Bx^n, \quad v = -nBx^{n-1}y, \tag{28a, b}$$

which give the flow field near the inclined plate.

Similarly, the substitution of equations (26) into equation (20) yields

$$\theta'' = \lambda\theta - \frac{1+n}{2}\eta\theta', \tag{29}$$

which, with boundary conditions (21b) and (22b), can be integrated numerically.

RESULTS AND DISCUSSION

Equations (24) and (25) with boundary conditions (21) and (22a) can be integrated numerically by the Runge–Kutta method with a systematic guessing of $\theta'(0)$ by the shooting technique. Integration has been carried out for the following two cases: (a) $\lambda = n = 0$ which corresponds to mixed convection from an isothermal vertical flat plate and (b) $\lambda = n = 1/3$ which corresponds to mixed convection from a flat plate with constant heat flux having an inclined angle of 45°. Results for $\theta(\eta)$ and $f'(\eta)$ for both aiding and opposing flows are shown in Figs. 2 and 3.

The results of greatest practical interest in a geothermal application are the thermal boundary layer thickness and the heat-transfer rate. Consider first the expression for the local surface heat flux along the inclined surface which can be computed from

$$q = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} = kA\left(\frac{B}{\alpha}\right)^{1/2} x^{(3\lambda-1)/2}[-\theta'(0)], \tag{30}$$

where the values of $[-\theta'(0)]$ as a function of Gr/Re for aiding and opposing flows are tabulated in Tables 1 and 2 respectively. Equation (30) shows that surface heat flux is constant for $\lambda = 1/3$. Equating equation (30) to the definition of h, i.e. $q = h(T_w - T_\infty)$ and rearranging, we have

$$\frac{Nu}{(RePr)^{1/2}} = [-\theta'(0)], \tag{31}$$

where $Nu \equiv hx/k$ and $Pr \equiv \nu/\alpha$. Equation (31) for aiding flows with $\lambda = 0$ and $\lambda = 1/3$ is plotted in Fig. 4 as a function of Gr/Re. It will be of interest to plot the corresponding expressions for pure free and pure forced convection in the same figure. For this purpose let us consider the case of forced convection where $Gr/Re = 0$. From Table 1, we have

$$\frac{Nu}{(RePr)^{1/2}} = 0.5641 \quad (\lambda = 0), \tag{32}$$

$$\frac{Nu}{(RePr)^{1/2}} = 0.8540 \quad (\lambda = 1/3),$$

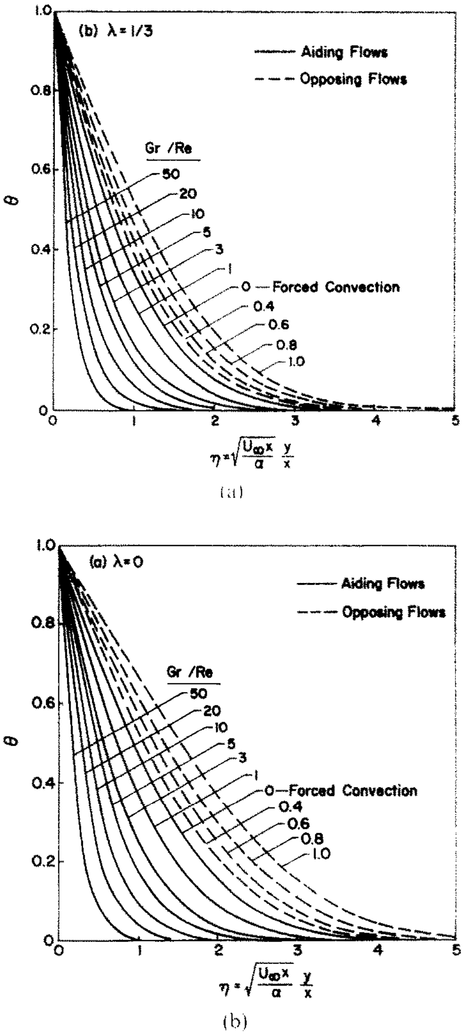

FIG. 2. Dimensionless temperature vs η for aiding and opposing flows (a) $\lambda = 0$ and (b) $\lambda = 1/3$.

Table 1. Values of $-\theta'(0)$ and η_T for aiding flows

	$\lambda = 0$		$\lambda = 1/3$	
Gr/Re	$-\theta'(0)$	η_T	$-\theta'(0)$	η_T
0	0.5641	3.6	0.8540	2.9
0.5	0.6473	3.3	0.9816	2.7
1.0	0.7205	3.1	1.093	2.5
3.0	0.9574	2.5	1.456	2.1
10.0	1.516	1.7	2.311	1.4
20.0	2.066	1.3	3.152	1.1

Table 2. Values of $-\theta'(0)$ and η_T for opposing flows

	$\lambda = 0$		$\lambda = 1/3$	
Gr/Re	$-\theta'(0)$	η_T	$-\theta'(0)$	η_T
0.2	0.5269	3.8	0.7970	3.0
0.4	0.4865	3.9	0.7351	3.2
0.6	0.4420	4.2	0.6671	3.3
0.8	0.3916	4.5	0.5903	3.5
1.0	0.3320	4.9	0.4999	3.8

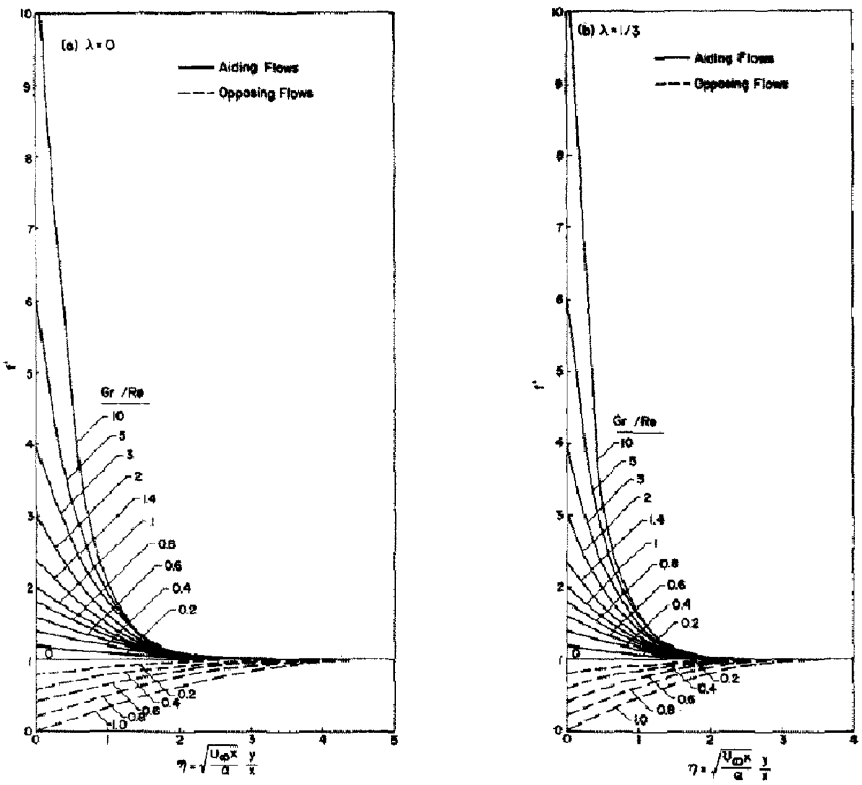

FIG. 3. Dimensionless velocity vs η for aiding and opposing flows (a) $\lambda = 0$ and (b) $\lambda = 1/3$.

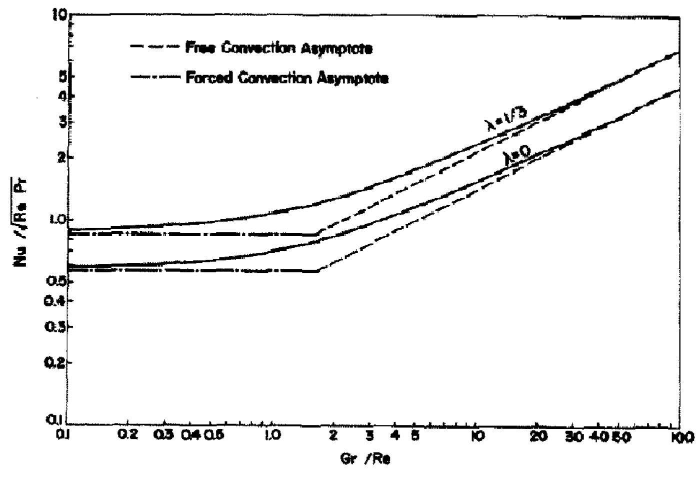

FIG. 4. Heat-transfer results for aiding flows.

which can also be obtained alternatively from the integration of equation (29) with $n = \lambda$. From the work of Cheng and Minkowycz [14], we have the following expressions for free convection about an inclined plate in a porous medium

$$\frac{Nu}{(Ra)^{1/2}} = 0.4440 \quad (\lambda = 0), \tag{33}$$

and

$$\frac{Nu}{(Ra)^{1/2}} = 0.6788 \quad (\lambda = 1/3),$$

which can be rewritten as

$$\frac{Nu}{(Re\,Pr)^{1/2}} = 0.444\left(\frac{Gr}{Re}\right)^{1/2}, \quad (\lambda = 0), \tag{34}$$

$$\frac{Nu}{(Re\,Pr)^{1/2}} = 0.6788\left(\frac{Gr}{Re}\right)^{1/2}, \quad (\lambda = 1/3),$$

where we have used the relation $Ra = Gr\,Pr$. Equations (32) and (34) are plotted as the forced and free convection asymptotes in Fig. 4. It is shown that the curves of $Nu/(RePr)^{1/2}$ for aiding flows lies above the asymptotes and that the maximum deviation from the asymptotes is 25–30% which occurs near $Gr/Re = 1.6$. The criteria for pure or mixed convection in a porous medium can be established if we follow the 5% deviation rule suggested by Sparrow *et al.* [1]. If this rule is applied to the local heat-transfer rate for aiding flows shown in Fig. 4, we have the following subdivisions

$$0 < Gr/Re < 0.15 \quad \text{forced convection,} \tag{35a}$$

$$0.15 < Gr/Re < 16 \quad \text{mixed flow,} \tag{35b}$$

$$16 < Gr/Re \quad \text{free convection.} \tag{35c}$$

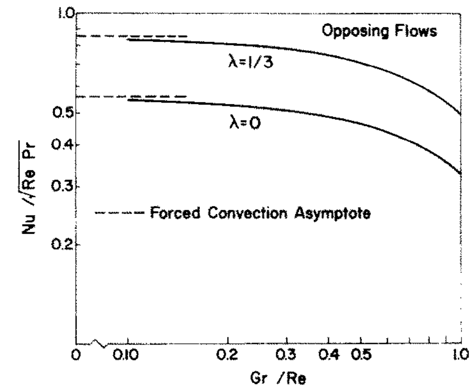

FIG. 5. Heat-transfer results for opposing flows.

The values of $Nu/(RePr)^{1/2}$ vs Gr/Re for opposing flows are plotted in Fig. 5 which shows that at small values of Gr/Re the curve approaches the forced convection asymptote. If the 5% deviation rule is again applied for opposing flows, it is found that equation (35a) is still valid but with equation (35b) replaced by

$$0.15 < Gr/Re. \tag{36}$$

The total surface heat-transfer rate for a flat plate with a length L and a width of S can be computed from

$$Q = S\int_0^L q(x)\,\mathrm{d}x, \tag{37}$$

which, after the substitution of equation (30) gives

$$Q = \frac{2SKA}{(3\lambda+1)}\left(\frac{B}{\alpha}\right)^{1/2}[-\theta'(0)]L^{(3\lambda+1)/2}. \tag{38}$$

Consider next the expression for thermal boundary-layer thickness. If η_T is the value of η at which $\theta(\eta)$ has a value of 0.01, we have, from equation (14)

$$\frac{\delta_T}{x} = \frac{\eta_T}{(RePr)^{1/2}}, \tag{39}$$

where the values of η_T are tabulated in Tables 1 and 2 for aiding and opposing flows. It will be of interest to show the values of $(RePr)^{1/2}\delta_T/x$ in the free and forced convection limits. This is done in Fig. 6 for aiding flow where the free convection asymptotes are given by Cheng and Mikowycz [14]

$$(RePr)^{1/2}\delta_T/x = \frac{6.31}{(Gr/Re)^{1/2}} \quad (\lambda = 0), \tag{40}$$

$$(RePr)^{1/2}\delta_T/x = \frac{5.50}{(Gr/Re)^{1/2}} \quad (\lambda = 1/3). \tag{41}$$

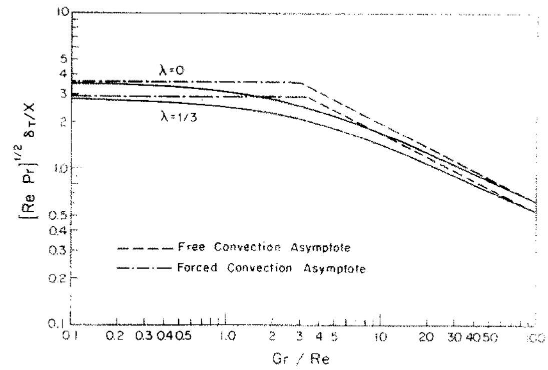

FIG. 6. Dimensionless boundary-layer thickness parameter for aiding flows.

To gain some feeling on the order of magnitude of various physical quantities in a geothermal application, consider a heated isothermal impermeable vertical surface, 1 × 1 km, embedded in an aquifer where a pressure gradient exists. If the temperature of the impermeable surface and the aquifer are at 215 and 15°C respectively and the pressure gradient is such that causes the groundwater moving upward vertically, the heat-transfer rate and the size of the hot water zone can be determined from Figs. 4 and 6. For numerical calculations, the following values of physical properties are used: $\beta = 1.8\times10^{-4}$°C, $\rho_\infty = 10^6$ g/m³, $C = 1$ cal/g°C, $\mu = 0.27$ g/s m, $k_m = 0.58$ cal/s°C m, and $K = 10^{-12}$ m². The results of the computations for U_∞ varying from 0.01 cm/h to 10 cm/h are plotted in Figs. 7 and 8 where it is shown that the total heat transfer rate increases from 20 to 120 MW while the boundary-layer thickness at 1 km decreases from 130 to 20 m.

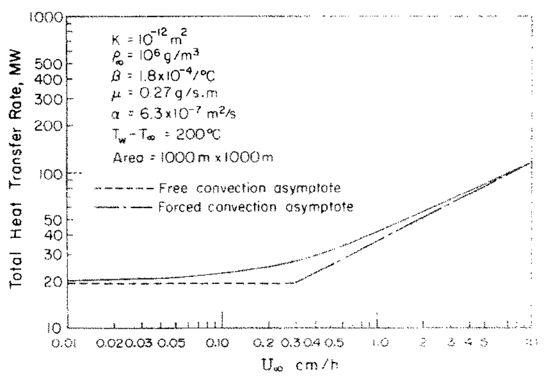

FIG. 7. Effects of U_∞ on total heat-transfer rate ($\lambda = 0$).

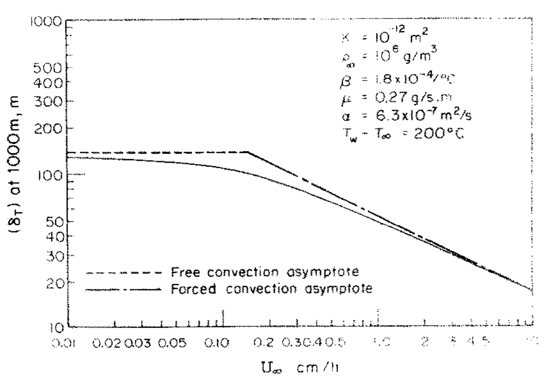

FIG. 8. Effects of U_∞ on boundary-layer thickness ($\lambda = 0$).

CONCLUDING REMARKS

The foregoing analysis is based on the boundary-layer approximations and neglecting the component of buoyancy force normal to the inclined surface. The latter assumption will break down when the inclined surface becomes horizontal. Thus, the analysis for mix convection about horizontal impermeable surfaces in a porous medium must be treated separately [15].

Acknowledgements—The author would like to take this opportunity to thank Mr. W. C. Chau for his assistance in the numerical computations. This study is part of the Hawaii Geothermal Project funded in part by the RANN program of the National Science Foundation of the United States (Grant No. GI-38319), the Energy Research and Development Administration of the United States [Grant No. E(04-3)-1093], and by the State and County of Hawaii.

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ECOULEMENTS MIXTES A COUCHE LIMITE LE LONG D'UNE SURFACE INCLINEE DANS UN MILIEU POREUX

Résumé—On analyse à partir des approximations de la couche limite le problème de la convection mixte sur des surfaces inclinées (dièdres) dans un milieu poreux saturé. Des solutions affines sont obtenues dans le cas particulier où la vitesse de l'écoulement libre et la distribution de température pariétale varient selon la même fonction puissance de la distance. On considère les écoulements favorables et défavorables. Le paramètre Gr/Re gouverne la convection mixte autour des surfaces inclinées. Des solutions numériques sont obtenues pour une plaque plane verticale isotherme et pour une plaque inclinée à flux constant avec un angle d'inclinaison égal à 45°. On donne les profils de température et de vitesse dans ces deux cas pour différentes valeurs de Gr/Re. Pour les écoulements favorables le transfert thermique approche les valeurs asymptotiques des convections forcées et naturelle lorsque Gr/Re approche les limites nulle et infinie. Les critères de la convection pure ou mixte autour des surfaces inclinées dans les milieux poreux sont précisés.

KOMBINIERTE FREIE UND ERZWUNGENE GRENZSCHICHTSTRÖMUNGEN AN GENEIGTEN OBERFLÄCHEN IN EINEM PORÖSEN MEDIUM

Zusammenfassung—Das Problem der kombinierten freien und erzwungenen Konvektion an geneigten Oberflächen (oder an keilförmigen Körpern) in einem gesättigten, porösen Medium wird anhand von Näherungslösungen der Grenzschichtgleichungen untersucht. Ähnlichkeitslösungen ergeben sich für den speziellen Fall, daß Geschwindigkeits- und Temperaturverteilung über der geneigten Oberfläche demselben Potenzgesetz gehorchen. Es werden Strömungen in Richtung des Auftriebs sowie entgegen dem Auftrieb betrachtet. Als beherrschender Parameter für die gemischte Konvektion an geneigten Oberflächen ergab sich der Quotient Gr/Re. Für die gemischte Konvektion an einer isothermen, vertikalen ebenen Platte sowie an einer um 45° geneigten Platte bei konstanter Wärmestromdichte werden numerische Lösungen sowie die Temperatur- und Geschwindigkeitsprofile für verschiedene Werte von Gr/Re angegeben. Bei Strömungen in Auftriebsrichtung nähert sich der Wärmeübergangskoeffizient asymptotisch den Werten bei erzwungener ($Gr/Re \to 0$) bzw. freier Konvektion ($Gr/Re \to \infty$). Für die reine und gemischte Konvektion an geneigten Oberflächen in porösen Medien werden Kriterien aufgestellt.

ТЕЧЕНИЕ В ПОГРАНИЧНОМ СЛОЕ НА НАКЛОННОЙ ПОВЕРХНОСТИ В ПОРИСТОЙ СРЕДЕ ПРИ СОВМЕСТНОЙ СВОБОДНОЙ И ВЫНУЖДЕННОЙ КОНВЕКЦИИ

Аннотация — В приближении пограничного слоя анализируется совместная свободная и вынужденная (смешанная) конвекция на наклонной поверхности (или клиньях) в насыщенной пористой среде. Получены автомодельные решения для случая, когда скорость набегающего потока и температура стенки наклонной поверхности изменяются по одной и той же степенной функции расстояния. Рассмотрены случаи спутного и обратного течения. Найдено, что отношение *Gr*/*Re* является параметром, определяющим процесс смешанной конвекции на наклонной поверхности в пористой среде. Получены численные решения для смешанной конвекции от изотермической плоской пластины, а также пластины, наклоненной под углом 45°, при постоянном тепловом потоке. Для этих двух случаев приведены профили температуры и скорости для различных значений отношения *Gr*/*Re*. Показано, что для спутного течения скорость теплообмена асимптотически приближается к значениям, характерным для вынужденной или свободной конвекции, по мере того, как отношение *Gr*/*Re* стремится к нулю или бесконечности. Установлены критерии для свободной и смешанной конвекции на наклонных поверхностях в пористых средах.